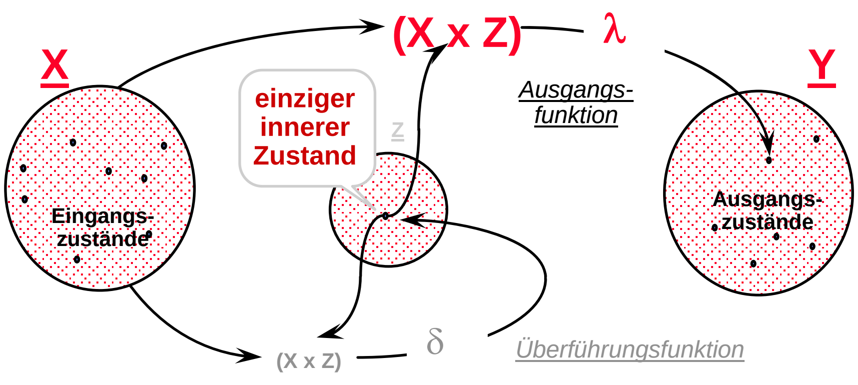
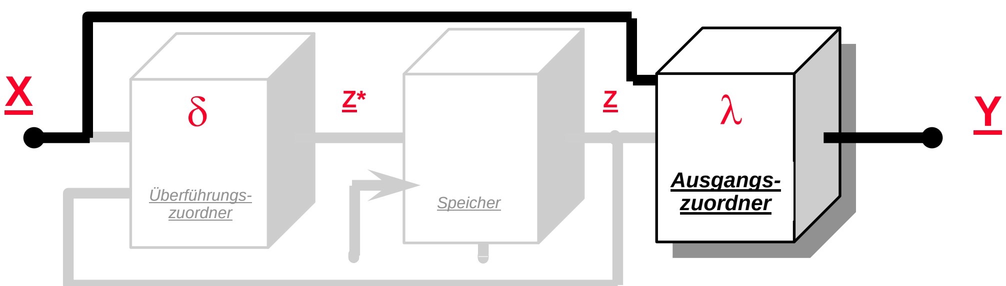
Buchstabenabbildungen

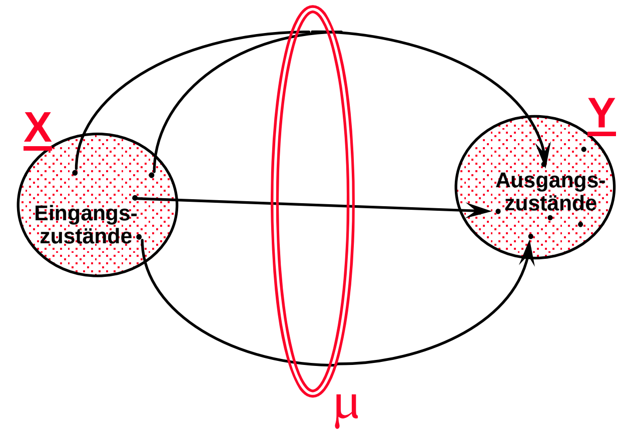
Digitale Systeme

Automat

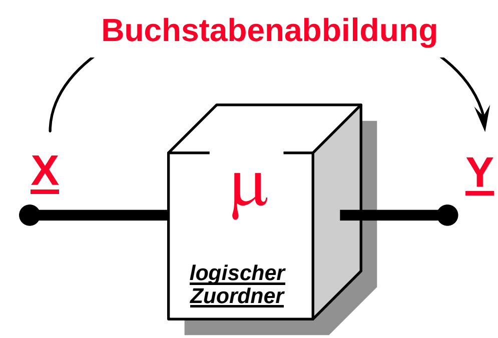


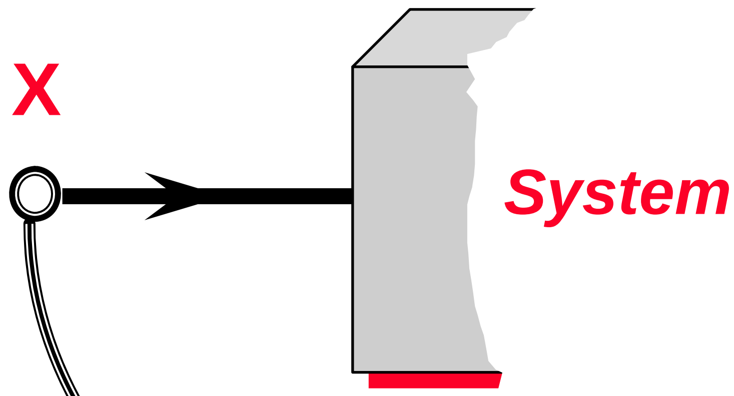


Kombinatorisches Netzwerk



Die Abbildungsvorschrift μ weist jedem Eingangsbuchstaben X_i eindeutig einen Ausgangsbuchstaben Y_j zu.



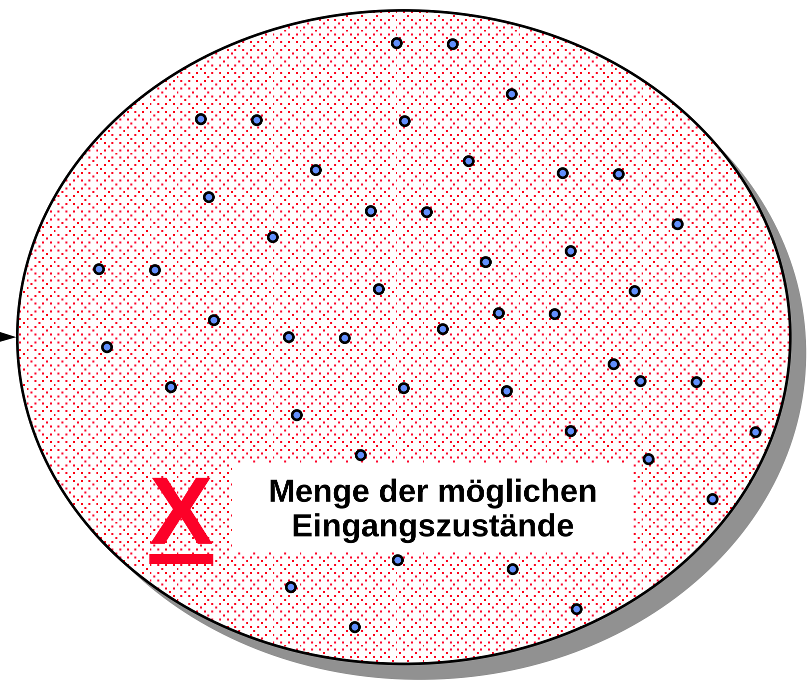


Jedem möglichen und sinnvollen Wert wird ein Punkt zugeordnet

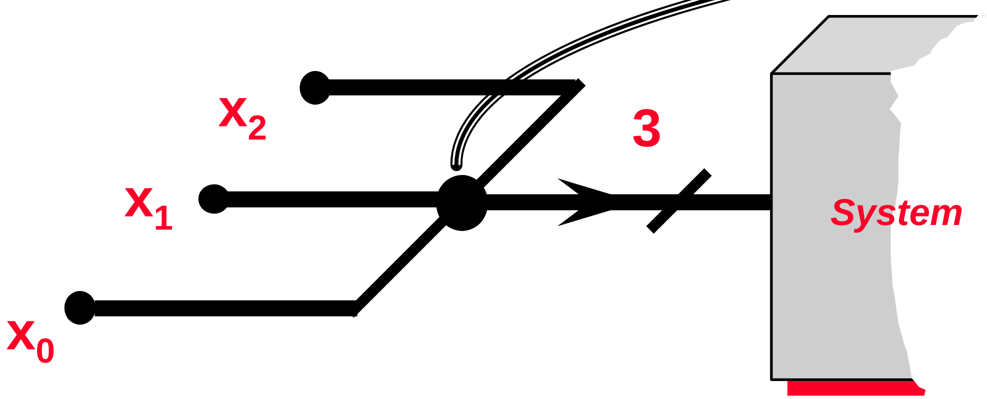
Code

Man unterscheidet:

Eingangscode
(inneren) Zustandscode
Ausgangscode

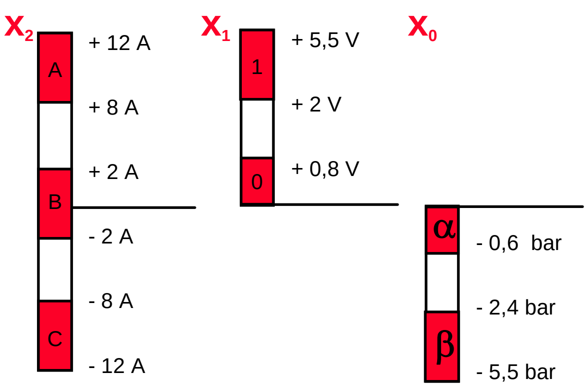


Variable- Buchstabe- Wort

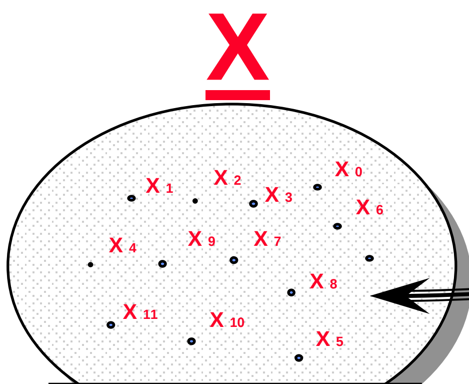


Variablen | Buchstaben

x_2	x_1	x_0	
A	0	α	X_0
A	0	β	X_1
A	1	α	X_2
A	1	β	X_3
B	0	α	X_4
B	0	β	X_5
B	1	α	X_6
B	1	β	X_7
C	0	α	X_8
C	0	β	X_9
C	1	α	X_{10}
C	1	β	X_{11}



Toleranzschwellen der (Eingangs)Signale auf den Variablen



Menge der möglichen Eingangszustände

Anzahl der möglichen Buchstaben

$$Az(\underline{X}) = \prod_{i=0}^{n-1} K_i$$

K_i = Wertigkeit der Variablen X_i

$$card(\underline{X}) = 3 * 2 * 2 = 12$$

Zur Erinnerung ! Das Dezimalsystem

n Stellen vor dem Komma **m Stellen nach dem Komma** **Basis des Zahlensystems (10)**

$$Z = 4523{,}2045$$

$$Z = 4*10^3 + 5*10^2 + 2*10^1 + 3*10^0 + 2*10^{-1} + 0*10^{-2} + 4*10^{-3} + 5*10^{-4}$$

4523 / 10	Rest
452	3
45	2
4	5
0	4

	2045*10
2	045
0	45
4	5
5	0

$$Z_{10} = \sum_{i=-4}^{3} x_i * 10^i$$

i	x_i
.	0
.	0
4	0
3	4
2	5
1	2
0	3
– 1	2
– 2	0
– 3	4
– 4	5
– 5	0
.	0
.	0

$$Z_a = \sum_{i=n-1}^{-m} x_i * a^i$$

Darstellung von Zahlen im Positionssystem

a = Basis des Zahlensystems
x_i = Koeffizient der Stelle a^i
i = Index
n = Anzahl der Stellen vor dem Komma
m= Anzahl der Stellen nach dem Komma
Das Komma steht nach der Stelle x_0

Die Umrechnung von einem Zahlensystem Z_a zum anderen Z_b erfolgt durch systematische Verschiebung der Zahlen im Zielzahlensystem über die Kommalinie.

Demonstrationsbeispiel

geg: Z_{10} **ges: Z_2**

42,625 **?**

42 / 2	Rest
21	0
10	1
5	0
2	1
1	0
0	1

	625*2
1	25
0	5
1	0

Z_2 = 101010,101

geg: Z_{10} **ges: Z_8**

42,625 **?**

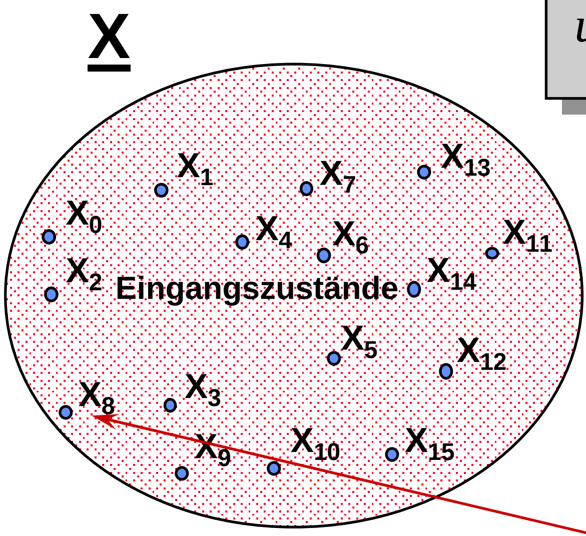
42 / 8	Rest
5	2
0	5

	625*8
5	0

Z_8 = 52,5

Z_2 = 101010,101

Z_8 = 52,5



Binärcode

Ordnungszahl

$$u = \sum_{0}^{3} x_i * 2^i$$

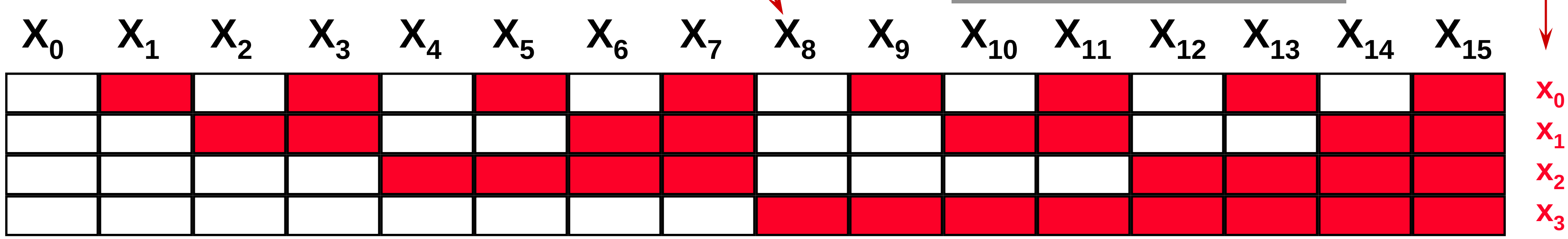
Binärcode

u	x_3	x_2	x_1	x_0	
0	0	0	0	0	X_0
1	0	0	0	1	X_1
2	0	0	1	0	X_2
3	0	0	1	1	X_3
4	0	1	0	0	X_4
5	0	1	0	1	X_5
6	0	1	1	0	X_6
7	0	1	1	1	X_7
8	1	0	0	0	X_8
9	1	0	0	1	X_9
10	1	0	1	0	X_{10}
11	1	0	1	1	X_{11}
12	1	1	0	0	X_{12}
13	1	1	0	1	X_{13}
14	1	1	1	0	X_{14}
15	1	1	1	1	X_{15}

Eingangsvariablen

Eingangsbuchstaben

Binärcode

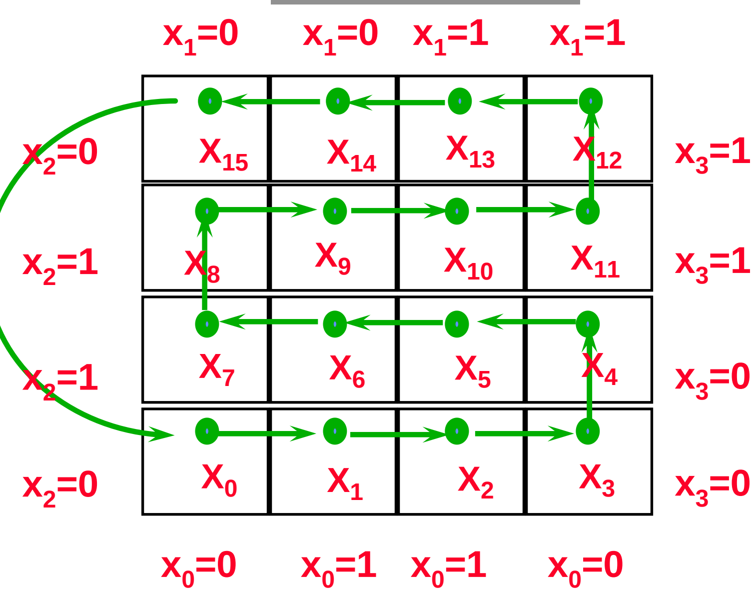


Graycode

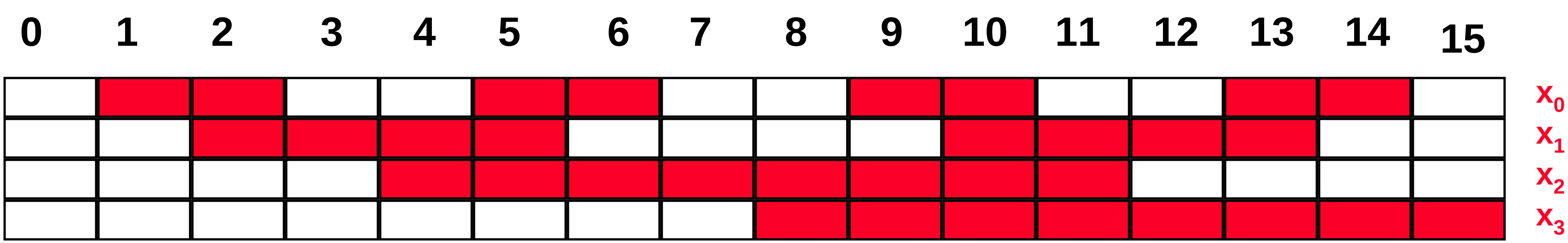
Graycode

u	x_3	x_2	x_1	x_0	
0	0	0	0	0	X_0
1	0	0	0	1	X_1
3	0	0	1	1	X_2
2	0	0	1	0	X_3
6	0	1	1	0	X_4
7	0	1	1	1	X_5
5	0	1	0	1	X_6
4	0	1	0	0	X_7
12	1	1	0	0	X_8
13	1	1	0	1	X_9
15	1	1	1	1	X_{10}
14	1	1	1	0	X_{11}
10	1	0	1	0	X_{12}
11	1	0	1	1	X_{13}
9	1	0	0	1	X_{14}
8	1	0	0	0	X_{15}

Karnaughplan



Benachbarte Buchstaben X_i und X_{i+1} unterscheiden sich lediglich durch eine Variable !



Einige Codierungen bei unteschiedlicher Motivation

2 aus 5 Code

	0	1	2	3	4	5	6	7	8	9
x_0	■	■						■	■	
x_1	■		■	■						■
x_2		■	■		■	■				
x_3				■	■		■	■		
x_4						■	■		■	■

1 aus 10 Code

	0	1	2	3	4	5	6	7	8	9
x_0	■									
x_1		■								
x_2			■							
x_3				■						
x_4					■					
x_5						■				
x_6							■			
x_7								■		
x_8									■	
x_9										■

m aus n - Codierungen erlauben

$$\binom{n}{m} = \frac{n!}{m! * (n-m)!}$$

Elemente.

Z.B. 2 aus 5

$$\frac{1 * 2 * 3 * 4 * 5 * 2}{1 * 2 * (1 * 2 * 3)}$$

$= 10$

Z.B. 1 aus 10 = 10

Hexadezimal - Code

Hex - Code					
u	x_3	x_2	x_1	x_0	X_i
0	0	0	0	0	X_0
1	0	0	0	1	X_1
2	0	0	1	0	X_2
3	0	0	1	1	X_3
4	0	1	0	0	X_4
5	0	1	0	1	X_5
6	0	1	1	0	X_6
7	0	1	1	1	X_7
8	1	0	0	0	X_8
9	1	0	0	1	X_9
10	1	0	1	0	X_A
11	1	0	1	1	X_B
12	1	1	0	0	X_C
13	1	1	0	1	X_D
14	1	1	1	0	X_E
15	1	1	1	1	X_F

$$u = \sum_{j=0}^{3} x_j 2^j$$

Einige Codes

BCD - Code

u	x_3	x_2	x_1	x_0	X_i
0	0	0	0	0	X_0
1	0	0	0	1	X_1
2	0	0	1	0	X_2
3	0	0	1	1	X_3
4	0	1	0	0	X_4
5	0	1	0	1	X_5
6	0	1	1	0	X_6
7	0	1	1	1	X_7
8	1	0	0	0	X_8
9	1	0	0	1	X_9
10	Pseudo-tetraden				X_{10}
11					X_{11}
12					X_{12}
13					X_{13}
14					X_{14}
15					X_{15}

3 - Exzeß - Code

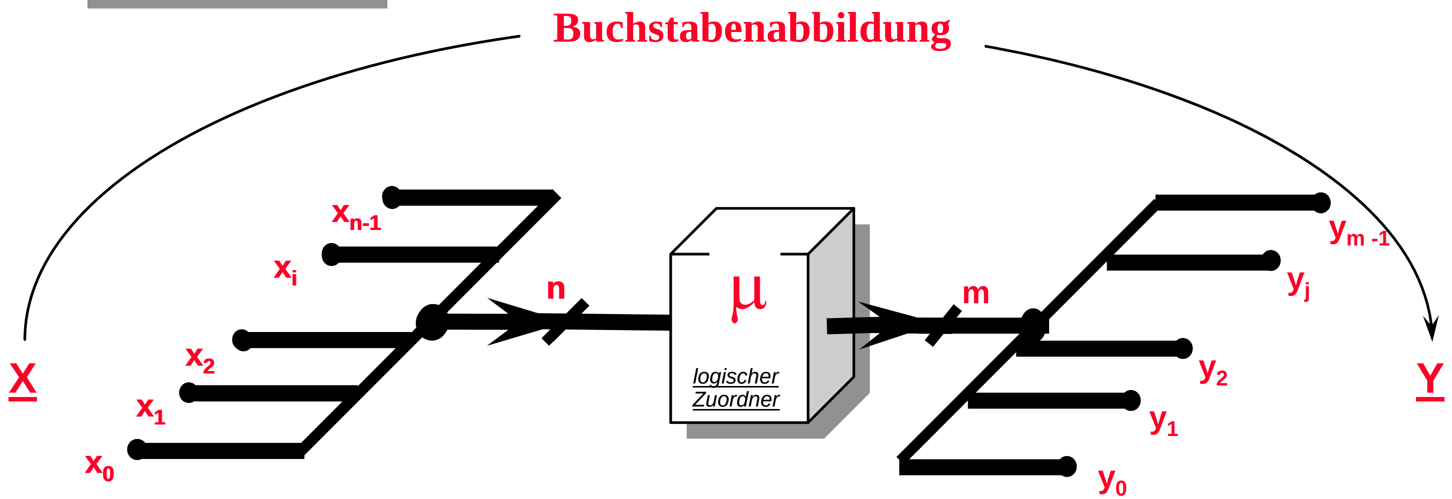
u	x_3	x_2	x_1	x_0	X_i
0	Pseudotetraden				
1					
2					
3	0	0	1	1	X_0
4	0	1	0	0	X_1
5	0	1	0	1	X_2
6	0	1	1	0	X_3
7	0	1	1	1	X_4
8	1	0	0	0	X_5
9	1	0	0	1	X_6
10	1	0	1	0	X_7
11	1	0	1	1	X_8
12	1	1	0	0	X_9
13	Pseudotetraden				
14					
15					

Aiken - Code

u	x_3	x_2	x_1	x_0	X_i
0	0	0	0	0	X_0
1	0	0	0	1	X_1
2	0	0	1	0	X_2
3	0	0	1	1	X_3
4	0	1	0	0	X_4
5	Pseudo-tetraden				
6					
7					
8					
9					
10					
11	1	0	1	1	X_5
12	1	1	0	0	X_6
13	1	1	0	1	X_7
14	1	1	1	0	X_8
15	1	1	1	1	X_9

X_i = codierter Wert
X_5 = im Binärcode codierte „5“
i ist im BCD - Code gleich u

Funktionsbündel



$$y_0 = f(x_0, x_1, \dots x_i \dots x_{n-1})$$

$$y_1 = f(x_0, x_1, \dots x_i \dots x_{n-1})$$

$$\vdots \qquad \vdots$$

$$y_j = f(x_0, x_1, \dots x_i \dots x_{n-1})$$

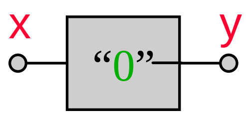
$$\vdots \qquad \vdots$$

$$y_{m-2} = f(x_0, x_1, \dots x_i \dots x_{n-1})$$

$$y_{m-1} = f(x_0, x_1, \dots x_i \dots x_{n-1})$$

Schaltfunktionen mit einer Variablen

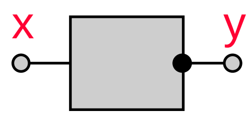
Nullfunktion



x	y
0	0
1	0

$$y = 0$$

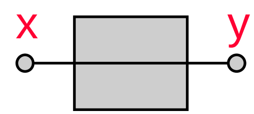
Negation



x	y
0	1
1	0

$$y = \bar{x}$$

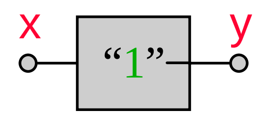
Identität



x	y
0	0
1	1

$$y \equiv x$$

Einsfunktion



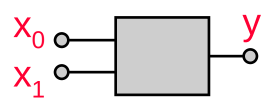
x	y
0	1
1	1

$$y = 1$$

Schaltfunktionen mit zwei Variablen

Tabelle | **Logiksymbol** | **Bezeichnung** | **Analytische Beschreibung**

x_1	x_0	$y_?$
0	0	
0	1	
1	0	
1	1	



?

$y = f\,(x_1 x_0)$

x_1	x_0	y_0	y_1	y_2	y_3	y_4	y_5	y_6	y_7	y_8	y_9	y_{10}	y_{11}	y_{12}	y_{13}	y_{14}	y_{15}
0	0	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1
0	1	0	0	1	1	0	0	1	1	0	0	1	1	0	0	1	1
1	0	0	0	0	0	1	1	1	1	0	0	0	0	1	1	1	1
1	1	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1

“0” $\quad \bar{x}_1 x_0 \quad x_1 \bar{x}_0 \quad x_0 \oplus x_1 \quad x_1 x_0 \quad x_0 \quad x_1 \quad x_1 \vee x_0$

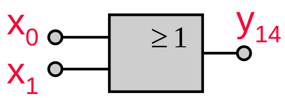
$\overline{x_0 \vee x_1} \quad \bar{x}_1 \quad \bar{x}_0 \quad \overline{x_0 x_1} \quad x_1 \otimes x_0 \quad \bar{x}_1 \vee x_0 \quad x_1 \vee \bar{x}_0$ “1”

Triviale Schaltfunktionen hängen nicht von allen **Eingangsvariablen** ab !

Nichttriviale Schaltfunktionen 1

ODER

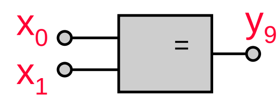
x_1	x_0	y_{14}
0	0	0
0	1	1
1	0	1
1	1	1



$$y_{14} = x_1 \vee x_0$$
$$= x_1 + x_0$$

Äquivalenz

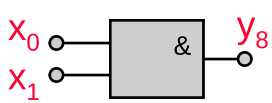
x_1	x_0	y_9
0	0	1
0	1	0
1	0	0
1	1	1



$$y_9 = x_1 \otimes x_0$$
$$= \bar{x}_1 \bar{x}_0 \vee x_1 x_0$$

UND

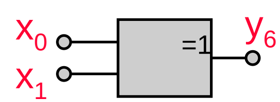
x_1	x_0	y_8
0	0	0
0	1	0
1	0	0
1	1	1



$$y_8 = x_1\ x_0$$
$$= x_1 * x_0$$
$$= x_1 \wedge x_0$$

Antivalenz

x_1	x_0	y_6
0	0	0
0	1	1
1	0	1
1	1	0

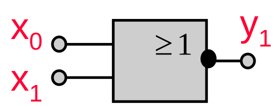


$$y_6 = x_1 \oplus x_0$$
$$= \bar{x}_1 x_0 \vee x_1 \bar{x}_0$$

Nichttriviale Schaltfunktionen 2

NOR

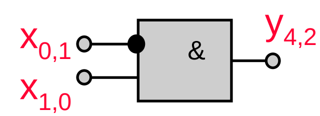
x_1	x_0	y_1
0	0	1
0	1	0
1	0	0
1	1	0



$$y_1 = \overline{x_1 \vee x_0} = \overline{x_1 + x_0} = \bar{x}_1 \, \bar{x}_0$$

Inhibition

x_1	x_0	y_2	y_4
0	0	0	0
0	1	1	0
1	0	0	1
1	1	0	0

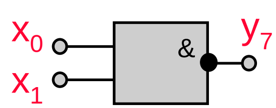


$$y_2 = \bar{x}_1 \, x_0$$

$$y_4 = x_1 \, \bar{x}_0$$

NAND

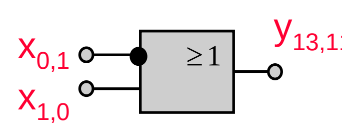
x_1	x_0	y_7
0	0	1
0	1	1
1	0	1
1	1	0



$$y_7 = \overline{x_1 \, x_0} = \overline{x_1 * x_0} = \overline{x_1 \wedge x_0}$$

Inklusion

x_1	x_0	y_{11}	y_{13}
0	0	1	1
0	1	1	0
1	0	0	1
1	1	1	1



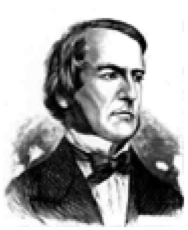
$$y_{11} = \bar{x}_1 \vee x_0$$

$$y_{13} = x_1 \vee \bar{x}_0$$

Die Boolesche Algebra

B (&, ∨, ¯, 0, 1)

UND → &
ODER → ∨
NICHT → ¯



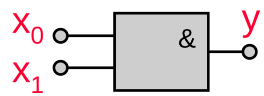
George Boole

* 2 Nov 1815 in Lincoln, Lincolnshire, England
† 8 Dec 1864 in Ballintemple, County Cork, Irland

Konjunktion

UND

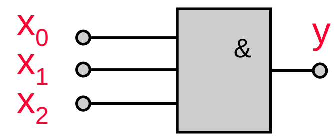
x_1	x_0	y
0	0	0
0	1	0
1	0	0
1	1	1



$$y = x_1\, x_0 = x_1 * x_0 = x_1 \wedge x_0$$

x_2	x_1	x_0	y
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	0
1	0	0	0
1	0	1	0
1	1	0	0
1	1	1	1

UND

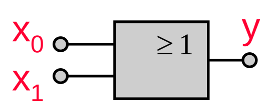


$$y = x_2 \wedge x_1 \wedge x_0 = x_2\, x_1\, x_0$$

Disjunktion

ODER

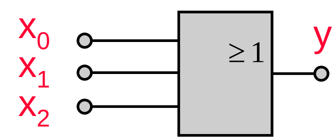
x_1	x_0	y
0	0	0
0	1	1
1	0	1
1	1	1



$$y = x_1 \vee x_0 = x_1 + x_0$$

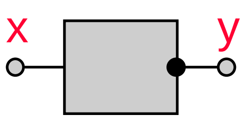
x_2	x_1	x_0	y
0	0	0	0
0	0	1	1
0	1	0	1
0	1	1	1
1	0	0	1
1	0	1	1
1	1	0	1
1	1	1	1

ODER



$$y = x_2 \vee x_1 \vee x_0 = x_2 + x_1 + x_0$$

Negation



x	y
0	1
1	0

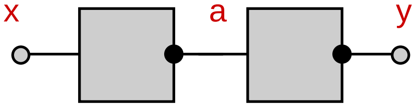
$$y = \overline{x}$$

Eigenschaften elementarer Schaltfunktionen

Digitale Systeme

NICHT

x	a	y
0	1	0
1	0	1



$a = \overline{x}$ $y = \overline{a}$

$\overline{0} = 1$ $\overline{1} = 0$ $\overline{\overline{a}} = a$

Augustus De Morgan

* : 27 June 1806 in Madura,
Madras Presidency, India
† : 18 March 1871 in London

Eigenschaft

Kommutativ	$a\,b = b\,a$	$a \vee b = b \vee a$
Assoziativ	$a\,(b\,c) = (a\,b)\,c = (a\,c)\,b$	$a \vee (b \vee c) = (a \vee b) \vee c = (a \vee c) \vee b$
Distributiv	$a\,(b \vee c) = a\,b \vee a\,c$	$a \vee b\,c = (a \vee b)\,(a \vee c)$
Idempotent	$a\,a = a$	$a \vee a = a$
Adjunktiv	$a\,(a \vee b) = a$	$a \vee a\,b = a$
Komplementär	$a \vee \overline{a} = 1$	$a\,\overline{a} = 0$

Wichtige Regeln !

De Morgansches Theorem: $\overline{a}\,\overline{b} = \overline{a \vee b}$ $\overline{a\,b} = \overline{a} \vee \overline{b}$

$a \vee 0 = a$ $a \vee \overline{a} = 1$ $a \vee 1 = 1$ $a\,0 = 0$ $a\,1 = a$ $a\,\overline{a} = 0$