# A conjecture on the s-rainbow polynomial

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Given a graph $G = (V, E)$ together with an edge-coloring $c : E \to \{1, \ldots, k\}$ with $k$ colors, we say that two vertices $s, t \in V$ have a rainbow-path when there exists an $s,t$-path where all edges on the path have different colors. If for a fixed vertex $s$, $c$ permits for all vertices $t$ a rainbow-path, we say that $c$ is an $s$-rainbow coloring. The $s$-rainbow polynomial $\rho(G, s, x)$ is a polynomial whose evaluation at $x \in \mathbb{N}$ gives the number of $s$-rainbow colorings of $G$ with $x$ colors. In 2016, Dod, Kischnick and Tittmann conjectured the following lower bound where $u$ is an articulation which separates $G$ in $G_1$ and $G_2$ with $s \in V(G_1)$:

$$\rho(G, s, x) \geq \rho(G_1, s, x - ecc(G_2, u)) \cdot \rho(G_2, u, x - ecc(G_1, s)).$$

In this talk we strengthen this conjecture to

$$\rho(G, s, x) \geq \rho(G_1, s, x - ecc(G_2, u)) \cdot \rho(G_2, u, x - d(s, u))$$

where $d(s, u)$ is the distance between $s$ and $u$ (both in $G$ and $G_1$) and prove this stronger variant for $d(s, u) \leq ecc(G_2, u) + 1$.