# Syllabus for the Master's Program: Applied Mathematics for Network and Data Sciences

## 1 Advanced Graph Theory and Network Algorithms

The course covers combinatorial aspects as well as applications of modern graph theory. The student will learn how to prove results in graph theory and how to apply graph theoretic concepts in different areas of application such as computer science, statistical physics, or communication technology. Topics:

- Connectivity in undirected graphs
- Graph isomorphism, graph invariants
- Distances in graphs
- Independence and domination
- Vertex and edge coloring of graphs
- Graph polynomials
- Graph classes: chordal graphs, partial k-trees
- Graph algorithms

## 2 Foundations of Modern Cryptography

Course objectives: Conveying a very deep understanding of the operation and safety of asymmetric cryptographic methods; imparting current research-related knowledge and methods; key skills; sharpening of program-ming skills. Topics:

- Computational number theory
- Public-key cryptosystems based on factoring and logarithms
- Cryptosystems based on NP-hard problems
- Digital signature schemes, DSS
- Elliptic curve cryptography.

# 3 Topics in Modern Analysis

Course objectives: Studies of basics of functional analysis, application to special problems of signal processing and data compression, ability to own scientific work based on in-depth mathematical skills Training of the following skills:

- systematisation and classification of mathematical problems in the mathematical field and in the field between mathematics and other disciplines,
- generalisation of basic mathematical correlations,
- proving (at a higher level of abstraction),
- application of methods which are based on the functional analysis to problems of science and technology and to current problems of computational mathematics,
- analysis and solution of typical application problems from image and data compression.

Topics:

- Basics of functional analysis: metric, norm, scalar product, Banach space, Hilbert space, orthonormal basis, orthogonal complement, separable Hilbert space, formal Fourier series, operators, properties of operators, application of the basic concepts to integral transforms, to differential and nonlinear equations
- Application to the Wavelet Transform: numerical approximation of functions, advantages and disadvantages of individual methods, Haar transformation, continuous wavelet transformation, discrete wavelet transformation – multiscale analysis.

# 4 Statistics and Probability Theory

The main objective is the acquirement of solid knowledge of probability theory and statistics. Students learn to handle various classes of stochastic processes and statistical models. Practical applications will be discussed in detail and implemented and solved using computerized methods. Based on that, students will gain a deep understanding of probability theory and statistics. Additionally, students acquire the abilities to comprehend practical problems conceptually, to structure, to classify and to solve them self-contained.

In this course measure-theoretical foundations of probability theory, probability- and moment-generating as well as characteristic functions, types of convergence, and limit theorems are introduced. The main focus lies on their applications in statistics (construction of statistical tests, asymptotic distributions). Theory will be made tangible by using stochastic simulations in the practical part of the course.

## 5 Communication Skills

This module includes I. Academic Standards, Writing and Presentations (in English) as well as II. Basic German (in German).

1. Upon successfully completing this module, students will be able to identify and describe in English key conventions and standards of academic/scholarly work and university correspondence/campus communication at a German university in general and within their course of studies in particular. They will be capable of constructively applying the acquired skills in their everyday life as a student.

2. Successful completion of this module will further provide students with the necessary skills to communicate effectively in German on an A1 level of the Common European Framework of Reference for Languages (CEFR) as well as to recognize and position intercultural differences. By means of these skills and insights, students are then better able to successfully navigate everyday situations and communication in their German surroundings.

## 6 Reliability of Communication Networks

The course Reliability of Communication Networks provides the student with the capability to establish mathematical models and to develop algorithmic solutions for reliability, reachability, and safety problems in communication networks. Topics:

- Monotone binary systems, structure function, path and cut set representations
- Reliability measures for of undirected graphs: all-terminal reliability, reliability polynomial, K-terminal reliability, edge decomposition, reductions, splitting techniques
- Reachability in digraphs, algebraic approaches to network reliability
- Algorithmic problems, complexity of network reliability calculations, simulation.

## 7 Simulation and Visualization

Students acquire in-depth knowledge in simulating processes with applications in life-sciences, engineering and economy. The mentioned contents qualify participants to address and solve applied problems using simulations techniques. Students will gain skills to categorize, analyze and implement complex problems. The abilities to assess simulation procedures and to pursue interdisciplinary approaches will be fostered.

Preparing a presentation about a self-chosen topic will deepen specific topics beyond the contents of the course and promote the students ability to independently advance in related topics.

In the tutorials, students will have the opportunity apply their acquired programming and software skills to independently work on simulation projects.

## 8 Computational Intelligence I

The course provides the basic principles and algorithms in CI. Particularly, neural networks for clustering and classification as well as Hebb learning are in the main focus. Completing the course, students are able to program basic models and to study their behavior. Topics:

- Biological neurons, perceptrons, multi-layer perceptrons,
- Hebbian learning, vector quantization.
- Machine Learning in MATLAB: programming of machine learning models in MATLAB,
- analysis of convergence behavior, exemplary applications.

## 9 Cryptanalysis

Course objectives: Conveying up-to-date knowledge and advanced methods on cryptanalysis; ability for independent acquisition of new knowledge; mastery of the international jargon. Topics:

- Attack scenarios
- Models and statements on the security of cryptographic methods Statistical Methods in cryptanalysis
- Linear and differential cryptanalysis, dictionary attacks
- Side channel attacks
- Password recovery (GPU-based, CUDA)
- Algebraic and number-theoretic methods
- Applications and real-world examples

## 10 Advanced Topics in Modern Cryptography

Course objectives: Dissemination of current research-related knowledge and advanced methods in the field of cryptography; ability to understand English-language papers and for the independent acquisition of new know-ledge; mastery of the international jargon of mathematics.

On a weekly basis, complex exercises are posed, the solutions of which are presented by the students in the seminar. The knowledge is deepened by programming exercises. Talks by students contribute to a strengthening of individual knowledge and competence.

## 11 Stochastic process with applications in signal processing

Goal of this course is to make the students familiar with the foundations of stochastic processes and their application in signal processing. Starting from the basics of probability theory and random variables, stochastic processes are introduced and their key features are studied with focus on signal processing applications like Markov processes which are widely used in autonomous systems.

The students are enabled to assess, analyze, design and specify as well as simulate signal processing systems dealing with stochastic processes. Topics:

- Basics of probability theory and random variables
- Continuous-time and discrete-time stochastic processes and their key parameters,
- Wide-sense-, strict-sense- and cyclo-stationary as well as ergodic stochastic processes,
- Gaussian and Markov processes,
- Generation and modeling of stochastic processes,
- Estimation and filtering of stochastic processes,
- Applications in signal processing, especially techniques used in autonomous, driving, speech and video processing.

## 12 Computational Intelligence II

The course provides advanced principles and algorithms in CI and discusses their realization. Additionally, students will start to study recent articles in the field, give short talks about recent developments and learn to communicate own ideas and problem solutions. Topics:

- Convergence and stability of algorithms,
- information theoretic learning,
- statistical learning theory and kernel methods,
- metric adaptation and feature selection,
- life-long learning, deterministic and simulated annealing,
- evolutionary algorithms, modern heuristics.

# 13 Scientfic Project

The main goal of this course is to empower students to do independent scientific work, which requires the development of methodological and analytic skills. The students learn to communicate mathematics effectively to mathematical and non-mathematical audiences in oral written, and multi-media form.

The students will learn:

- general principles of mathematical problem solving,
- ways to attack hard (computational intractable) problems,
- combining analytical and computational methods,
- Presentation of mathematical results.

# 14 Selective Modules

Students of the master's course are required to finish at least two out of the following modules.

## 14.1 Discrete Structures

Objectives: Teaching basic mathematical structures; teaching basic ways of thinking and methods of discrete mathematics; developing the ability to abstract; introduction to research-related topics; teaching the key skills tenacity and perseverance; ability to prove mathematical hypotheses.

The topics are taken from (but not restricted to) set systems, independence systems, matroids, projective and com-binatorial geometry, antimatroids, greedoids, convex geometry, lattic-es, semirings, transversals, latin squares, combinatorial designs; Ramsey theory.

On a weekly basis, exercises are posed, whose solutions are pre-sented by the students during the seminar.

## 14.2 Signals and Systems

The aim of the course is to provide knowledge for the analysis, description, classification and transformation of random variables and to chance processes, to estimate its characteristic parameters and its effect on and influence by linear/nonlinear and/or systems time-variant / invariant systems.

The course starts with an introduction and deepening of the foundations of probability theory and description of random processes. Based on this, one- and multi-dimensional transformations of random variables and the generation of random variables and processes considered to chance with desired properties. This includes an introduction to the properties and application of Markov chains.

The second part of the lecture contains methods to estimate the cur-rent implementation of a random variable and the properties of the underlying random process. This includes, among other things, the motivation and derivation of the MAP- and ML-estimation rules and the Cramer-Rao bound for estimating the predictive accuracy.

Contents of the third part of the lecture is the processing of random processes by means of filters with the objective of interference, in-terpolation and estimate of random variables. It is the derivation and application of the matched filter and Wiener filter, associated adaptive filter algorithms (LMS, RLS) and the Kalman-filter.

## 14.3 Programming in C++

C++ is a widely used in technical applications, object-oriented pro-gram-ming language. The students will be able to apply the concepts of object-oriented programming in programming with the specific programming language C++. To learn the STL, BOOST library, the cross-platform Qt framework and the Intel Threading Building Blocks of the company and how to apply. Similarly, the students of computer science and mathematics are thereby able to collaborate interdisciplinary with computer scientists and engineers in large pro-gramming projects. It sharpens their abstraction ability and the ability to conceptual, logical, structural and algorithmic thinking encouraged.

- Program development environments for C + +,
- Extensions of C in the sense of a "better C" areas such as names, functions with default arguments, ...
- class as an abstract data type, defining classes, creating objects, operators, and objects / assignment operator, operator overload-ing,
- Class variables and class function
- aggregation and association of objects
- inheritance: single inheritance with access rights, choice of meth-ods, polymorphism, multiple inheritance with the various struc-tures and Zugriffspgfaden,
- exception handling,
- Generic programming - class templates,
- Friend functions and classes
- containers and algorithms of the C + + standard library objects simpler and polymorphic types,
- the Qt library and its use for graphical interfaces,
- the BOOST libraries, TBB and SFML.

## 14.4 Selected Topics in Computational Statistics

Students expand their knowledges in statistical methods, in theory and practice. In particular, the students acquire the ability to independently plan studies and carry out the statistical analysis on the computer.

The most essential statistical procedures are discussed in theory and application (Data-handling, descriptive statistics, one- and two-sample t-tests, ANOVA, and respective non-parametric counter-parts, goodness-of-fit tests, linear models, power-calculations, confidence intervals, multiple testing, etc.). Furthermore, a selection of more sophisticated statistical methods will be discussed (e.g. Bootstrap, Approximate Bayesian Computation, Likelihood-methods).

## 14.5 Discrete Optimization

Students who have successfully completed this course should

- have acquired the knowledge about problems, models and solution techniques of discrete optimization,
- be able to find exact and heuristic approaches to special discrete optimization problems,
- have learned to develop algorithms for the solution of practical discrete optimization problems and to estimate the quality of obtained solutions.

The module includes basic topics of discrete and especially the combinatorial optimization:

- models of discrete optimization (problems and their mathematical representation),
- exact solutions (cutting plane algorithms, method of branch-and-bound, branch-and-cut algorithms),
- heuristic procedures (greedy algorithms and local search methods, meta strategies, genetic algorithms),
- criteria of quality and complexity,
- independent systems and matroids.

## 14.6 Mathematical Logic

After completing the module, students will be able to formalize complex problems using propositional and predicate logic, and to solve these problems algorithmically. Through the study of English literature as well as exercises and contributed talks, students will be able to communicate their own ideas and solutions using the mathematical jargon. They will be able to formulate, prove or disprove mathematical hypotheses. Topics:

- propositional logic,

- predicate logic,
- Automated Theorem Proving,
- Logic Programming,
- Undecidability.

## 14.7 Selected Topics in Discrete Mathematics

Goals of the module:

- communicating current and research-related topics,
- working with scientific original papers,
- mediation of mathematical terminology in oral and written proficiency for independent acquisition of new knowledge,
- ability to formulate, prove or disprove mathematical hypotheses,
- deep investigation of a branch of discrete mathematics, e.g., enumerative combinatorics, extremal combinatorics or randomized algorithms.

On a weekly basis, complex exercises are posed, the solutions of which are presented by the students in the seminar. The tasks for each student include a short presentation of 45 minutes duration on current topics.

## 14.8 Programming Project

The module will enable the student to

- improve programming skills,
- to develop the ability to use documentation tools,
- to perform independent scientific work and to work in interdisciplinary teams,
- to communicate mathematical content in speech and writing

In the accompanying seminar, students regularly report on the progress of the project.

## 14.9 Digital Communications

Goal of this course is to make the students familiar with the principles of modern digital data transmission systems. Starting from the information theoretic basics of digital communication, the components of digital communication systems are studied with focus on digital transmission and multiple access schemes such as OFDM.

The students are enabled to assess, analyze, design and specify as well as simulate digital communication systems.

- Principles of digital transmission, channel models and multiple access schemes (TDMA, CDMA, OFDM)
- Major digital modulation schemes and their performance for var-ious channel models
- Transmit and receive techniques used to increase transmission diversity and minimize interference
- Overview and comparison of major digital transmission systems (data rates, spectral and power efficiency)
- Forthcoming developments, especially techniques used in 4th generation mobile communication standard and planned for 5th generation standard.

## 14.10 Discrete Mathematical Modeling

The student will learn to create own mathematical models for different practical application scenarios that lead to problems in discrete mathematics. This includes a proper understanding of the given problem, finding appropriate parameters and objective functions, choosing the right model, developing algorithms, evaluating and interpreting results. The covered topics include:

- problems in modeling complex systems: defining borders of a system, choosing the right model, finding appropriate measures,
- developing algorithms for exact and approximate solutions, investigating computational complexity,
- exploring the differences between simulation of systems and analytical solutions
- investigating examples from different fields of application: communication networks, computer systems, information processing.

## 14.11 Digital Video Analysis

Goal of this course is to make the students familiar with the foundations of digital image and video processing and their application in video analysis. Starting from the physical basics and key components of digital image and video recording and compression systems, standard im-age and video processing tasks and the used algorithms are studied first. Based on these advanced techniques which are applied especially in video forensics and autonomous systems are introduced.

The students are enabled to assess, analyze, design and specify as well as simulate image and video processing systems. Topics:

- Physical basics of image representation and recording
- Key components of digital image and video processing and compression systems

- Standard image manipulations applying e.g. point and morphological operations, affine transformations, contrast adjustment
- Image and video analysis and feature detection, classification and representation using e.g. Fourier- and Wavelet-transformation, integral images and self-learning classification techniques
- Applications of video analysis, especially forensic video analysis, face detection and recognition in videos and autonomous driving

## 14.12 Advanced Topics in Computer Science

The goal of this course is to equip students with mathematical techniques and practical skills that are useful for the design and analysis of algorithms, for the investigation of large data sets, and for the analysis and optimization of computer and information systems. This class also provides the student with the theoretical foundation necessary to effectively perform research in computer science. Topics:

- Modeling structures: finding appropriate data structures and representations.
- Basic methods of analysis of algorithms, time and space requirements, limits of computation.
- In addition a selection of topics from complexity theory, finite automata, language theory, algebraic methods in computer science, mathematical logic, and cryptography is presented.
- Applications in science and industries.

## 14.13 Selected Topics in Computational Mathematics

This course provides advanced principles and knowledge about mathematical approaches and discusses their numerical realization. Additionally, students will start to study recent articles in the field, give short talks about recent developments and learn to communicate own ideas and problem solutions. Topics are dimensionality estimation in data sets, sparse models, robust estimators of parameters, non-Euclidean dissimilarities, independent component analysis and blind source separation, recent topics.