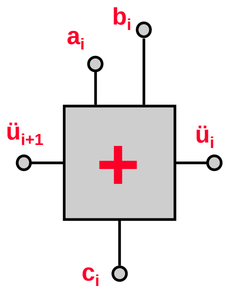
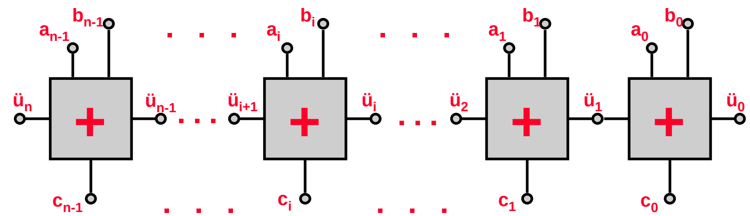
# Basissysteme am Beispiel eines Adders




| $ü_i$ | $b_i$ | $a_i$ | $ü_{i+1}$ | $c_i$ |
|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 0 | 1 |
| 0 | 1 | 0 | 0 | 1 |
| 0 | 1 | 1 | 1 | 0 |
| 1 | 0 | 0 | 0 | 1 |
| 1 | 0 | 1 | 1 | 0 |
| 1 | 1 | 0 | 1 | 0 |
| 1 | 1 | 1 | 1 | 1 |

$$ü_{i+1} = \bar{ü}_i b_i a_i \vee ü_i \bar{b}_i a_i \vee ü_i b_i \bar{a}_i \vee ü_i b_i a_i$$

$$c_i = \bar{ü}_i \bar{b}_i a_i \vee \bar{ü}_i b_i \bar{a}_i \vee ü_i \bar{b}_i \bar{a}_i \vee ü_i b_i a_i$$

$ü_{i+1}$ (KV-Diagramm; $b_i$ über den beiden rechten Spalten, $a_i$ unter den beiden mittleren Spalten, $ü_i$ an der oberen Zeile)

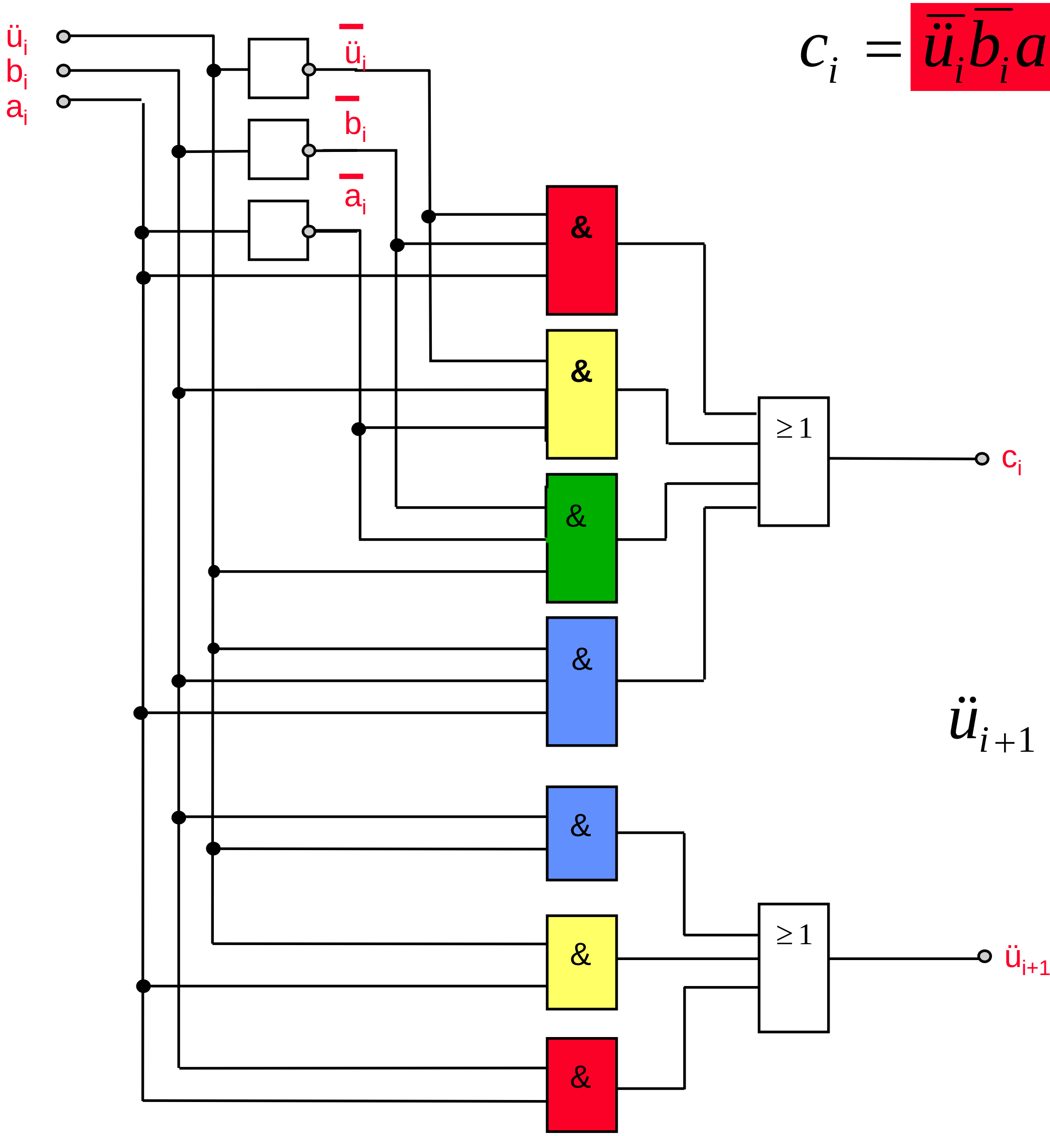
| | | | | |
|---|---|---|---|---|
| 0 | 1 | 1 | 1 | $ü_i$ |
| 0 | 0 | 1 | 0 | |

$c_i$ (KV-Diagramm; $b_i$ über den beiden rechten Spalten, $a_i$ unter den beiden mittleren Spalten, $ü_i$ an der oberen Zeile)

| | | | | |
|---|---|---|---|---|
| 1 | 0 | 1 | 0 | $ü_i$ |
| 0 | 1 | 0 | 1 | |

$$ü_{i+1} = ü_i b_i \vee ü_i a_i \vee b_i a_i$$

# Logikplan des Volladders




$$c_i = \bar{ü}_i \bar{b}_i a_i \vee \bar{ü}_i b_i \bar{a}_i \vee ü_i \bar{b}_i \bar{a}_i \vee ü_i b_i a_i$$

$$ü_{i+1} = ü_i b_i \vee ü_i a_i \vee b_i a_i$$

# NAND

**1** DN oder KDN erzeugen

$$c_i = \overline{\ddot{u}}_i \overline{b}_i a_i \vee \overline{\ddot{u}}_i b_i \overline{a}_i \vee \ddot{u}_i \overline{b}_i \overline{a}_i \vee \ddot{u}_i b_i a_i$$

$$\ddot{u}_{i+1} = \ddot{u}_i b_i \vee \ddot{u}_i a_i \vee b_i a_i$$

**2** Doppelte Negation

$$c_i = \overline{\overline{\overline{\ddot{u}}_i \overline{b}_i a_i \vee \overline{\ddot{u}}_i b_i \overline{a}_i \vee \ddot{u}_i \overline{b}_i \overline{a}_i \vee \ddot{u}_i b_i a_i}}$$

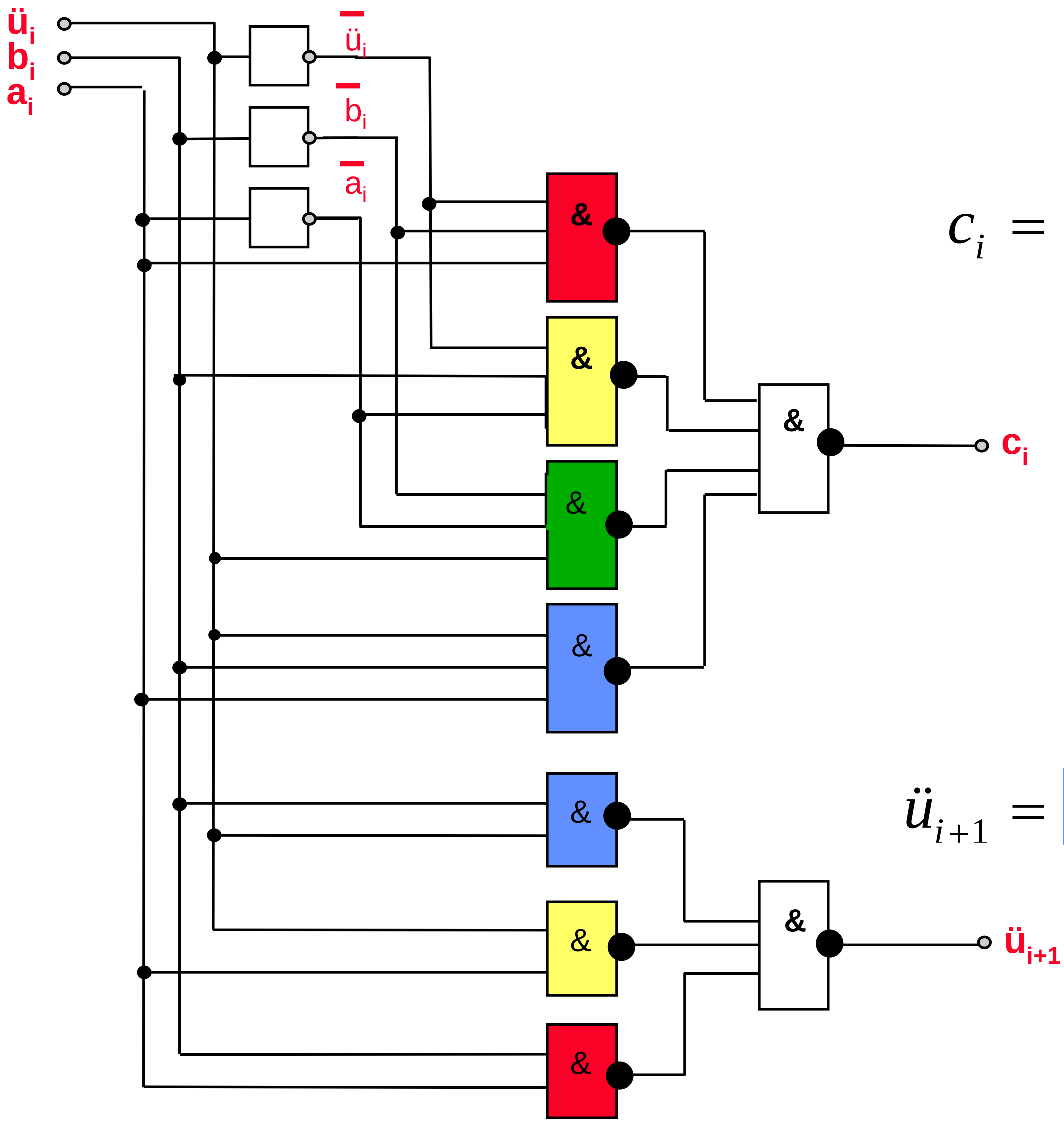
$$\ddot{u}_{i+1} = \overline{\overline{\ddot{u}_i b_i \vee \ddot{u}_i a_i \vee b_i a_i}}$$

**3** Auflösen der unteren Negation

$$c_i = \overline{\overline{\overline{\ddot{u}}_i \overline{b}_i a_i} \quad \overline{\overline{\ddot{u}}_i b_i \overline{a}_i} \quad \overline{\ddot{u}_i \overline{b}_i \overline{a}_i} \quad \overline{\ddot{u}_i b_i a_i}}$$

$$\ddot{u}_{i+1} = \overline{\overline{\ddot{u}_i b_i} \quad \overline{\ddot{u}_i a_i} \quad \overline{b_i a_i}}$$

# Logikplan mit NAND

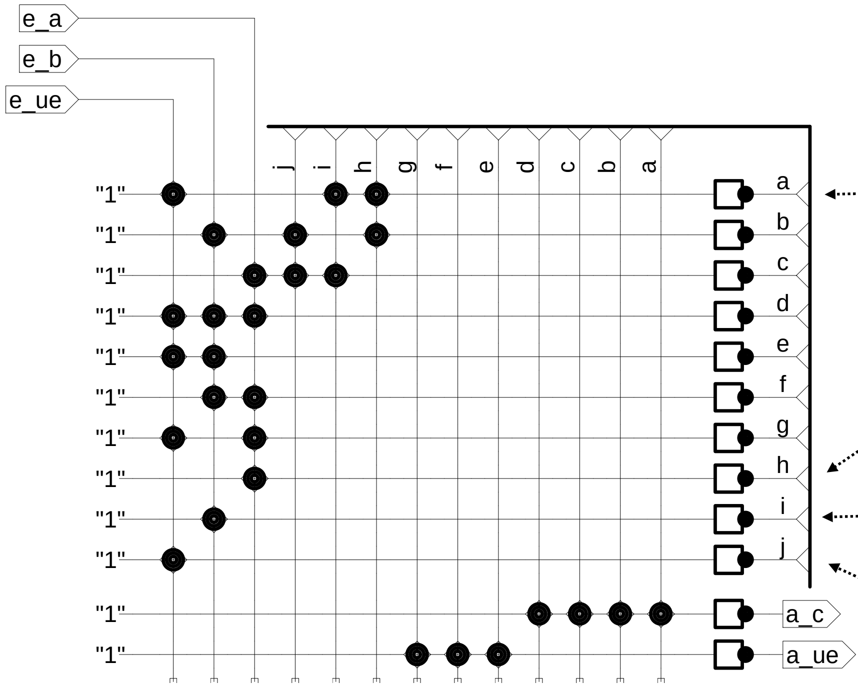



$$c_i = \overline{\overline{\bar{ü}_i \bar{b}_i a_i}\;\overline{\bar{ü}_i b_i \bar{a}_i}\;\overline{ü_i \bar{b}_i \bar{a}_i}\;\overline{ü_i b_i a_i}}$$

$$ü_{i+1} = \overline{\overline{ü_i b_i}\;\overline{ü_i a_i}\;\overline{b_i a_i}}$$

# Logikplan mit NAND - Array

**Digitale Systeme**

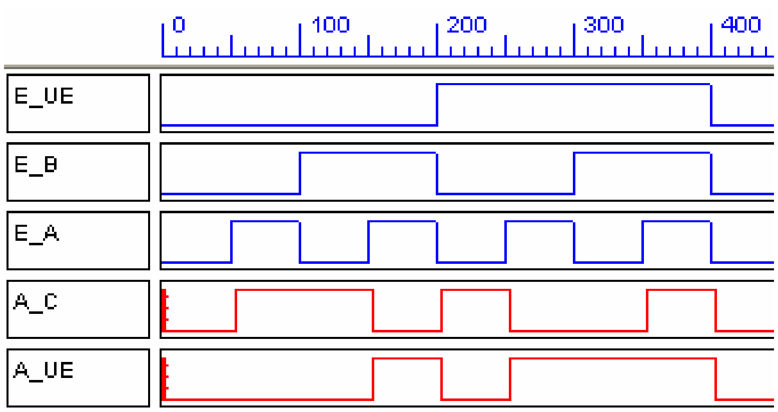


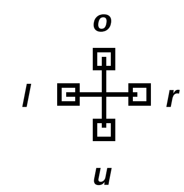

$a = \overline{e\_ue * \overline{e\_b} * \overline{e\_a}}$

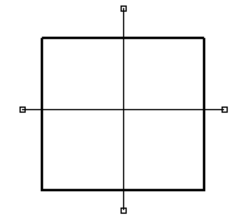
$h = \overline{e\_a}$

$i = \overline{e\_b}$

$j = \overline{e\_ue}$

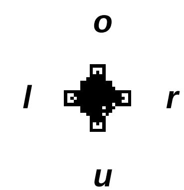


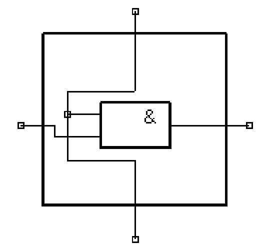



$u = o$

$r = l$




$u = o$

$r = l * o$

## NOR (Erzeugung der konjunktiven Normalformen)

**1** KKN oder KN erzeugen

| $ü_i$ | $b_i$ | $a_i$ | $ü_{i+1}$ | $c_i$ |
|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 0 | 1 |
| 0 | 1 | 0 | 0 | 1 |
| 0 | 1 | 1 | 1 | 0 |
| 1 | 0 | 0 | 0 | 1 |
| 1 | 0 | 1 | 1 | 0 |
| 1 | 1 | 0 | 1 | 0 |
| 1 | 1 | 1 | 1 | 1 |

$$\overline{ü_{i+1}} = \overline{ü}_i\overline{b}_i\overline{a}_i \vee \overline{ü}_i\overline{b}_i a_i \vee \overline{ü}_i b_i\overline{a}_i \vee ü_i\overline{b}_i\overline{a}_i$$

$$\overline{c_i} = \overline{ü}_i\overline{b}_i\overline{a}_i \vee \overline{ü}_i b_i a_i \vee ü_i\overline{b}_i a_i \vee ü_i b_i\overline{a}_i$$

$$\overline{ü_{i+1}} = \overline{ü}_i\overline{b}_i\overline{a}_i \vee \overline{ü}_i\overline{b}_i a_i \vee \overline{ü}_i b_i\overline{a}_i \vee ü_i\overline{b}_i\overline{a}_i$$

$$\overline{ü_{i+1}} = \overline{ü}_i\overline{b}_i \vee \overline{ü}_i\overline{a}_i \vee \overline{b}_i\overline{a}_i$$

$$\overline{\overline{c_i}} = \overline{\overline{ü}_i\overline{b}_i\overline{a}_i \vee \overline{ü}_i b_i a_i \vee ü_i\overline{b}_i a_i \vee ü_i b_i\overline{a}_i}$$

$$c_i = (ü_i \vee b_i \vee a_i)(ü_i \vee \overline{b}_i \vee \overline{a}_i)(\overline{ü}_i \vee b_i \vee \overline{a}_i)(\overline{ü}_i \vee \overline{b}_i \vee a_i)$$

$$ü_{i+1} = (ü_i \vee b_i)(ü_i \vee a_i)(b_i \vee a_i)$$

$$c_i = (\ddot{u}_i \vee b_i \vee a_i)(\ddot{u}_i \vee \overline{b_i} \vee \overline{a_i})(\overline{\ddot{u}_i} \vee b_i \vee \overline{a_i})(\overline{\ddot{u}_i} \vee \overline{b_i} \vee a_i)$$

$$\ddot{u}_{i+1} = (\ddot{u}_i \vee b_i)(\ddot{u}_i \vee a_i)(b_i \vee a_i)$$

**2** Doppelte Negation der konjunktiven Normalformen

$$\overline{\overline{c_i}} = \overline{\overline{(\ddot{u}_i \vee b_i \vee a_i)(\ddot{u}_i \vee \overline{b_i} \vee \overline{a_i})(\overline{\ddot{u}_i} \vee b_i \vee \overline{a_i})(\overline{\ddot{u}_i} \vee \overline{b_i} \vee a_i)}}$$

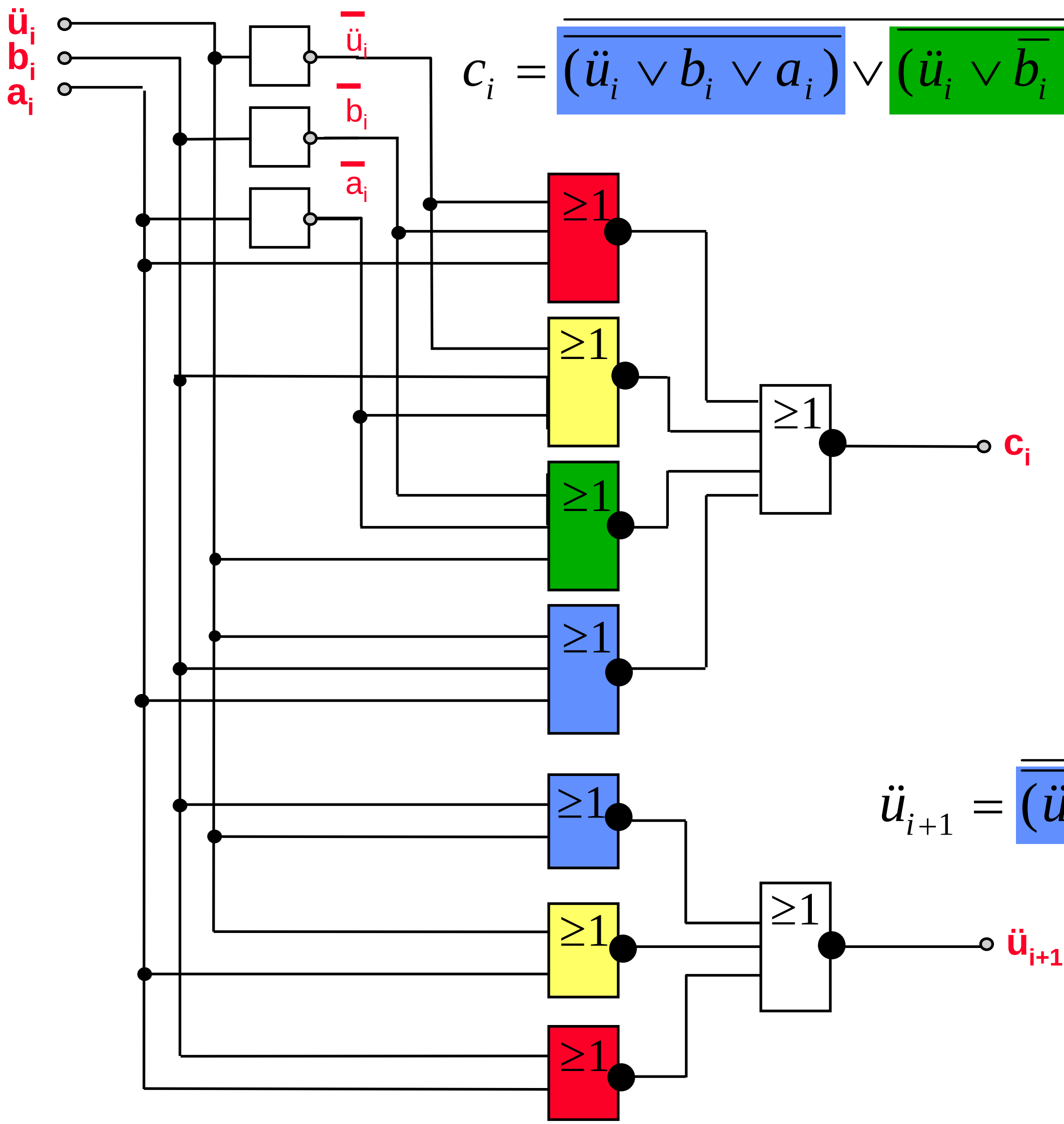
$$\overline{\overline{\ddot{u}_{i+1}}} = \overline{\overline{(\ddot{u}_i \vee b_i)(\ddot{u}_i \vee a_i)(b_i \vee a_i)}}$$

**3** Auflösen der unteren Negation

$$c_i = \overline{\overline{(\ddot{u}_i \vee b_i \vee a_i)} \vee \overline{(\ddot{u}_i \vee \overline{b_i} \vee \overline{a_i})} \vee \overline{(\overline{\ddot{u}_i} \vee b_i \vee \overline{a_i})} \vee \overline{(\overline{\ddot{u}_i} \vee \overline{b_i} \vee a_i)}}$$

$$\ddot{u}_{i+1} = \overline{\overline{(\ddot{u}_i \vee b_i)} \vee \overline{(\ddot{u}_i \vee a_i)} \vee \overline{(b_i \vee a_i)}}$$

## Logikplan mit NOR




$$c_i = \overline{\overline{(\ddot{u}_i \vee b_i \vee a_i)} \vee \overline{(\ddot{u}_i \vee \overline{b}_i \vee \overline{a}_i)} \vee \overline{(\overline{\ddot{u}}_i \vee b_i \vee \overline{a}_i)} \vee \overline{(\overline{\ddot{u}}_i \vee \overline{b}_i \vee a_i)}}$$

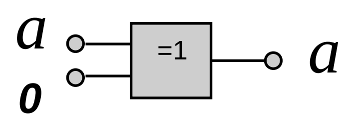
$$\ddot{u}_{i+1} = \overline{\overline{(\ddot{u}_i \vee b_i)} \vee \overline{(\ddot{u}_i \vee a_i)} \vee \overline{(b_i \vee a_i)}}$$

## Die alternative Normalform

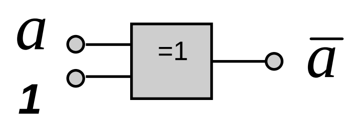
**0**

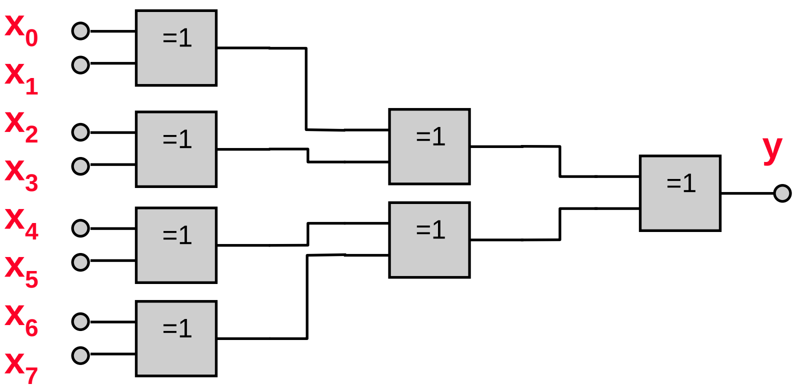
**Die ANF enthält lediglich Antivalenz und UND. Keinesfalls eine Negation!**

$$y = a \oplus 0 \quad = a\overline{0} \vee \overline{a}0 \quad = a1 \vee a0 \quad = a$$




$$y = a \oplus 1 \quad = a\overline{1} \vee \overline{a}1 \quad = a0 \vee \overline{a}1 \quad = \overline{a}$$




**y = 0 wenn die Anzahl der Einsen an $X_i$ gerade ist.**

**Sonst y = 1**

**Damit kann man in kanonischen disjunktiven Normalformen das ODER durch die Antivalenz ersetzen.**

## Analytische Umwandlung

$$\ddot{u}_{i+1} = \overline{\ddot{u}}_i b_i a_i \vee \ddot{u}_i \overline{b}_i a_i \vee \ddot{u}_i b_i \overline{a}_i \vee \ddot{u}_i b_i a_i$$

**1. Ersetzen des ODER durch Antivalenz**

$$= \overline{\ddot{u}}_i b_i a_i \oplus \ddot{u}_i \overline{b}_i a_i \oplus \ddot{u}_i b_i \overline{a}_i \oplus \ddot{u}_i b_i a_i$$

**2. Ersetzen der Negation durch Antivalenz “1”**

$$= (\ddot{u}_i \oplus 1) b_i a_i \oplus \ddot{u}_i (b_i \oplus 1) a_i \oplus \ddot{u}_i b_i (a_i \oplus 1)_i \oplus \ddot{u}_i b_i a$$

**3. Anwenden des Distributivgesetzes**

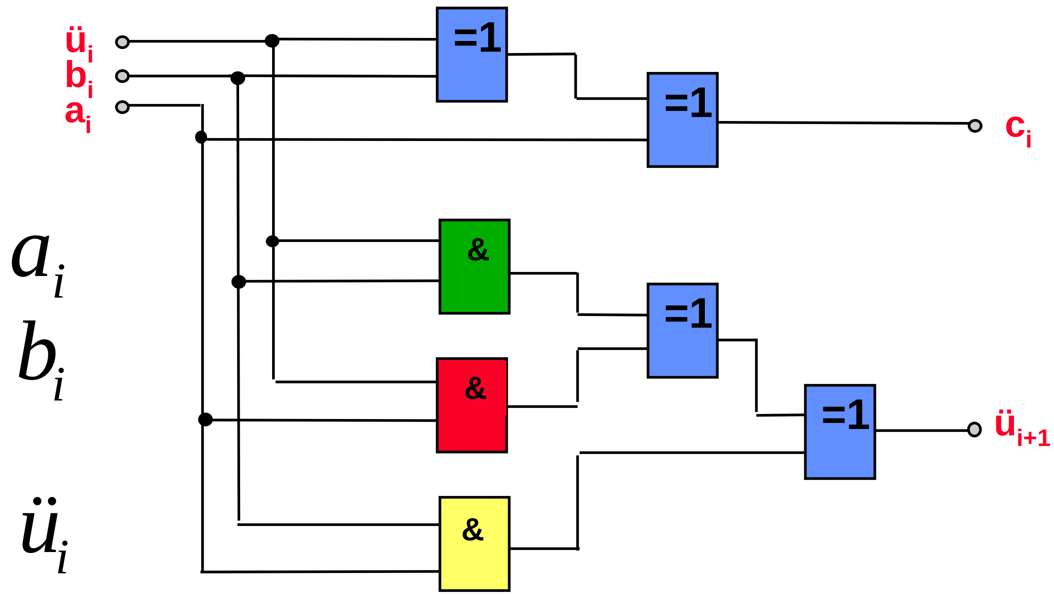
$$= \cancel{\ddot{u}_i b_i a_i} \oplus b_i a_i \oplus \cancel{\ddot{u}_i b_i a_i} \oplus \ddot{u}_i a_i \oplus \cancel{\ddot{u}_i b_i a_i} \oplus \ddot{u}_i b_i \oplus \cancel{\ddot{u}_i b_i a}$$

**4. Streichen paariger Terme**

$$= b_i a_i \oplus \ddot{u}_i a_{i_i} \oplus \ddot{u}_i b_i$$

# Tabellenmethode

| $ü_i$ | $b_i$ | $a_i$ | $c_i$ | | | | |
|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | | | | |
| 0 | 0 | 1 | 1 | / | | | |
| 0 | 1 | 0 | 1 | | / | | |
| 0 | 1 | 1 | 0 | / | / | | |
| 1 | 0 | 0 | 1 | | | / | |
| 1 | 0 | 1 | 0 | / | | / | |
| 1 | 1 | 0 | 0 | | / | / | |
| 1 | 1 | 1 | 1 | / | / | / | / |




$$c_i = a_i \oplus b_i \oplus ü_i$$

$$ü_{i+1} = b_i a_i \oplus ü_i a_i \oplus ü_i b_i$$