Rechenregeln für Differentialoperatoren

$\nabla(\varphi_1+\varphi_2)=\nabla\varphi_1+\nabla\varphi_2$	$grad\ (\varphi_1+\varphi_2)=grad\ \varphi_1+grad\ \varphi_2$
$\nabla(\varphi_1\varphi_2)=\varphi_2\nabla\varphi_1+\varphi_1\nabla\varphi_2$	$grad\ (\varphi_1\varphi_2)=\varphi_2\ grad\ \varphi_1+\varphi_1\ grad\ \varphi_2$
$\nabla(f(\varphi))=\frac{df}{d\varphi}(\varphi)\nabla\varphi$	$grad\ (f(\varphi))=\frac{df}{d\varphi}(\varphi)\ grad\ \varphi$
$\nabla\cdot(\vec{u}_1+\vec{u}_2)=\nabla\cdot\vec{u}_1+\nabla\cdot\vec{u}_2$	$div\ (\vec{u}_1+\vec{u}_2)=div\ \vec{u}_1+div\ \vec{u}_2$
$\nabla\cdot(\varphi\,\vec{u})=\vec{u}\cdot\nabla\varphi+\varphi\ \nabla\cdot\vec{u}$	$div\ (\varphi\ \vec{u})=\vec{u}\cdot grad\ \varphi+\varphi\ div\ \vec{u}$
$\nabla\cdot(\vec{u}_1\times\vec{u}_2)=\vec{u}_2\cdot(\nabla\times\vec{u}_1)-\vec{u}_1\cdot(\nabla\times\vec{u}_2)$	$div\ (\vec{u}_1\times\vec{u}_2)=\vec{u}_2\cdot\ rot\ \vec{u}_1-\vec{u}_1\cdot\ rot\ \vec{u}_2$
$\nabla\cdot(\nabla\times\vec{u})=0$	$div\ rot\ \vec{u}=0$
$\nabla\times(\vec{u}_1+\vec{u}_2)=\nabla\times\vec{u}_1+\nabla\times\vec{u}_2$	$rot\ (\vec{u}_1+\vec{u}_2)=rot\ \vec{u}_1+rot\ \vec{u}_2$
$\nabla\times(\vec{u}_1\times\vec{u}_2)=(\vec{u}_2\cdot\nabla)\vec{u}_1-\vec{u}_2\nabla\cdot\vec{u}_1$ $+\vec{u}_1(\nabla\cdot\vec{u}_2)-(\vec{u}_1\cdot\nabla)\vec{u}_2$	$rot\,(\vec{u}_1\times\vec{u}_2)=(\vec{u}_2\cdot grad)\,\vec{u}_1-\vec{u}_2\ div\ \vec{u}_1$ $+\vec{u}_1\ div\ \vec{u}_2-(\vec{u}_1\cdot grad)\,\vec{u}_2$
$\nabla\times(\nabla\varphi)=\vec{0}$	$rot\ grad\ \varphi=\vec{0}$
$\nabla\times(\nabla\times\vec{u})=\nabla(\nabla\cdot\vec{u})-\nabla\cdot\nabla\vec{u}$ $=\nabla(\nabla\cdot\vec{u})-\triangle\ \vec{u}$	$rot\ rot\ \vec{u}=grad\,(div\ \vec{u})-div\ grad\ \vec{u}$
$\nabla\times(\varphi\vec{u})=\varphi\ \nabla\times\vec{u}-\vec{u}\times\nabla\varphi$	$rot\,(\varphi\vec{u})=\varphi\ rot\ \vec{u}-\vec{u}\times grad\ \varphi$
$\triangle\varphi=\nabla\cdot\nabla\varphi=\varphi_{xx}+\varphi_{yy}+\varphi_{zz}$	$\triangle\varphi=div\ grad\ \varphi=\varphi_{xx}+\varphi_{yy}+\varphi_{zz}$
$\triangle(\varphi_1\varphi_2)=\varphi_2\triangle\varphi_1+2\nabla\varphi_1\nabla\varphi_2+\varphi_1\triangle\varphi_2$	
$\triangle(f(\varphi))=f'(\varphi)\triangle\varphi+f''(\varphi)(\nabla\varphi)^2$ $\triangle\ \vec{u}=\nabla(\nabla\cdot\vec{u})-\nabla\times(\nabla\times\vec{u})$	$div\ grad\ \vec{u}=grad\,(div\ \vec{u})-rot\ rot\ \vec{u}$

Integralsätze

1. $\int_V grad\ u\ dV=\oint_{\partial V}u\ dS$	$V\subset\mathbb{R}^3\ bzw.\ V\subset\mathbb{R}^2$
Satz von Gauß: 2. $\int_V div\ \overrightarrow{u}\ dV=\oint_{\partial V}\overrightarrow{u}\cdot\overrightarrow{dS}$	$V\subset\mathbb{R}^3\ bzw.\ V\subset\mathbb{R}^2$
3. $\int_V rot\ \overrightarrow{u}\ dV=-\oint_{\partial V}\overrightarrow{u}\times\overrightarrow{dS}$	$V\subset\mathbb{R}^3\ bzw.\ V\subset\mathbb{R}^2$
Satz von Stokes: 4. $\int_F rot\ \overrightarrow{u}\cdot\overrightarrow{dS}=\oint_{\partial F}\overrightarrow{u}\cdot\overrightarrow{dx}$	$F-Fl\ddot{a}che\ im\ \mathbb{R}^3$
5. $\int_F grad\ u\times\overrightarrow{dS}=-\oint_{\partial F}u\ dx$	$F-Fl\ddot{a}che\ im\ \mathbb{R}^3$
Greensche Formeln: 6. $\int_V u\triangle v\ dV=\oint_{\partial V}u\frac{\partial u}{\partial\overrightarrow{n}}dS-\int_V grad\ u\cdot grad\ v\ dV$	$V\subset\mathbb{R}^3\ bzw.\ V\subset\mathbb{R}^2$
7. $\int_V u\ div\ \overrightarrow{v}\ dV=\oint_{\partial V}u\ \overrightarrow{v}\cdot\overrightarrow{dS}-\int_V grad\ u\cdot\overrightarrow{v}\ dV$	$V\subset\mathbb{R}^3\ bzw.\ V\subset\mathbb{R}^2$