About some Unified Approaches to Graph Polynomials

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We introduce graph polynomials as ordinary generating functions for the number of (spanning, induced) subgraphs of a given graph. We denote the family of all finite undirected graphs by $\mathcal{G}$. Let p be a *graph property* (usually a graph invariant), which means that p is a mapping $p : \mathcal{G} \to \{\text{true}, \text{false}\}$. A graph property can be given as a *graph predicate* like 'is Hamiltonian,' 'is connected,' 'is planar,' or 'has a perfect matching.'

Let r be a given positive integer and p a graph property. For $i = 1, \ldots, r$, let $k_i : \mathcal{G} \to \mathbb{N}$ be a function that assigns to any finite graph a nonnegative integer (usually a numerical graph invariant), for instance the number of vertices, edges, components, or spanning trees. A general definition of a graph polynomial is now

$$f(G; x_1, \ldots, x_r) = \sum_{H \leq G} [p(H)] \prod_{i=1}^{r} x_i^{k_i(H)}.$$

The relation "$\leq$" can be read as "is subgraph of" or "is spanning subgraph of" or "is induced subgraph of," depending on the application that we intend.

We will discuss questions concerning calculation, comparison, complexity, classification, and application of graph polynomials.