Lösungen der Hausaufgaben MB-Master

A) Differentialrechnung für Funktionen mehrerer Variabler

1a) $D_f = \left\{(x,y)^T \in \mathbb{R}^2 | (x \leq 0 \wedge y \leq 0) \vee (x \geq 0 \wedge y \geq 0)\right\}$; $\quad W_f = [0;\infty)$

1b) $D_f = \left\{(x,y)^T \in \mathbb{R}^2 | x \neq -y\right\}$; $\quad W_f = \mathbb{R} \setminus \{0\}$

2) $D_f = \left\{(x,y)^T \in \mathbb{R}^2 | 1 \leq x^2 + y^2 < 2\right\}$; $\quad$ unstetig bei $x^2 + y^2 = 2$

3) $\mathrm{u}_x = \frac{x}{\sqrt{x^2+y^2+z^2}}$; $\quad \mathrm{u}_y = \frac{y}{\sqrt{x^2+y^2+z^2}}$ $\quad \mathrm{u}_z = \frac{z}{\sqrt{x^2+y^2+z^2}}$

$\Longrightarrow (u_x)^2 + (u_y)^2 + (u_z)^2 = \frac{x^2}{x^2+y^2+z^2} + \frac{y^2}{x^2+y^2+z^2} + \frac{z^2}{x^2+y^2+z^2} = 1$

4) $y' = -\frac{x^2-ay}{y^2-ax}$

5a) $f_x = z^x(\frac{1}{x} + \ln\frac{x}{y} \cdot \ln z)$ $\quad$ 5b) $f_z = yz^{y-1} + e^{z^2x} \cdot 2zx$

6a) Maximum in $(-3;3;-1)^T$ $\quad$ 6b) Minimum in $(2;1;-10)^T$ und Maximum in $(-2;-1,10)^T$

7) $|\triangle E| \lessapprox 528350\frac{V}{m}$; $\quad \frac{|\triangle E|}{E} \lessapprox 0.02976$

8) $\left|\frac{\triangle V}{V}\right| \lessapprox 0.02$

9) $\nabla f = (2x, 2y, 2z)^T$; $\quad \frac{df}{d\underline{l}}|_P = 0.8165$

10) $x + 2y - z = 2.5$

B) Bereichsintegrale

1) $I = \int\limits_0^1 \int\limits_{-\sqrt{y}}^{\sqrt{y}} f(x,y)dxdy + \int\limits_1^4 \int\limits_{y-2}^{\sqrt{y}} f(x,y)dxdy$

2) $I = -2$

3) $m = \frac{32}{3}ME$; $\quad x_s = 1.5$; $\quad y_s = 1.5$

4) $x_s = y_s = 0.58$

5) $J_z = \frac{1}{6}$

6) $V = 81\pi VE$

7) $x_s = y_s = 0$; $\quad z_s = -\frac{3}{8}R$; $\quad V = \frac{2}{3}\pi R^3 VE$

8) $I = 168\pi VE$

C) Kurvenintegrale

1a) $s = 2.3LE$; $\quad$ 1b) $s = 16LE$

2) $I = 0.5$

3) $W = \frac{1}{3}$

4) $I = -a^2 b\frac{\pi}{2}$

5) wegunabhängig, Potential: $F = xyz + \frac{1}{3}y^3 + C$; $\quad I = \frac{28}{3}$

6) wegunabhängig, Potential: $F = -xy + \frac{1}{2}z^2 + C$; $\quad I = \frac{1}{2}\pi^2$

D) Vektoranalysis

1)$(4;4;4)^T$, Zuwachs $= 12$

2) $(\frac{1}{3},\ -\frac{2}{3},\ -\frac{2}{3})^T$

3) $div\underline{v}\,|_P = -3$, $\quad rot\underline{v}\ |_P = (-6;0;0)^T$

4) $a = 4; b = 2; c = -1$

6) $I = 8$

7) $I = 20\pi$

8) $V = 13\ (VE/ZE)$